

CLAIMS

1. A method wherein mesh insensitive structural stress σ_s in a localized fatigue-prone weld region of a structure is calculated from a finite element model of said structure by:

5 identifying local elements for structural stress extraction, wherein said local elements lie adjacent to said weld region;

determining nodal displacements and nodal force and moment vectors for said local elements from said finite element model;

10 converting selected ones of said nodal force and moment vectors to sectional force vectors n and moment vectors m , wherein said conversion is performed in a work equivalent manner with respect to said nodal displacements determined for said nodal force and moment vectors; and

15 calculating said structural stress from said sectional force vectors n and moment vectors m .

20 2. A method as claimed in claim 1 wherein said conversion is performed such that a quantity of work corresponding to said nodal displacements and nodal force and moment vectors is equivalent to a quantity of work for said nodal displacements and said sectional force and moment vectors n and m .

25 3. A method as claimed in claim 1 wherein said nodal force and moment vectors are converted to sectional force vectors n and moment vectors m with a mapping function corresponding to said finite element model.

4. A method as claimed in claim 3 wherein said mapping function is selected such that said sectional force vector n has units of force per unit length and said sectional moment vector m has units of moment per unit length.

5. A method of analyzing structural stress σ_s in a localized fatigue-prone region of a structure from a representation of the structure, said method comprising:

determining a through-thickness stress distribution $\sigma_x(y)$ along a selected cross section of said structure, wherein said localized region lies adjacent to said selected cross section, and wherein said stress distribution $\sigma_x(y)$ is determined from said representation of said structure;

determining a first component σ_M of said structural stress σ_s in said localized region by performing an operation having a result substantially equivalent to a result of the following first integration

$$\sigma_M = \frac{1}{t} \int_0^t \sigma_x(y) dy$$

where $\sigma_x(y)$ represents said through-thickness stress distribution and t corresponds to the thickness of said structure in said selected cross section;

determining a second component σ_B of said structural stress σ_s in said localized region by performing an operation having a result substantially equivalent to a solution of the following equation for σ_B

$$\left(\frac{t^2}{2}\right)\sigma_M + \left(\frac{t^2}{6}\right)\sigma_B = \int_0^t \sigma_x(y)y dy + \delta \int_0^t \tau_{xy}(y) dy$$

where y corresponds to a distance from $y=0$ to a material point of interest along said selected cross section, t corresponds to the thickness of said structure in said selected cross section, δ is a value defined in said representation of said structure, $\sigma_x(y)$ represents said through-thickness stress distribution, and $\tau_{xy}(y)$ represents a through-thickness shear stress distribution of said structure; and

calculating said structural stress σ_s by combining said first component σ_M of said structural stress and said second component σ_B of said structural stress.

- 5 6. A method of analyzing structural stress as claimed in claim 5 wherein said second component σ_B of said structural stress σ_s in said localized region is determined by defining the following relation where the shear stress distribution in the region is negligible:

$$\delta \int_0^t \tau_{xy}(y) dy = 0$$

- 10 7. A method of analyzing structural stress as claimed in claim 6 wherein said shear stress distribution $\tau_{xy}(y)$ is determined from said structural representation.

- 15 8. A method of analyzing structural stress as claimed in claim 4 wherein said selected cross section of said structure is defined by a thickness t and includes a partial thickness fatigue crack extending a distance t_1 from a surface of said localized region, and wherein said method comprises:

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 20 determining said first and second components σ_M , σ_B of said structural stress σ_s in said localized region by performing respective operations having results substantially equivalent to respective results of four equations, where a first equation of said four equations defines a sub-component σ_m' of the structural stress σ_s , a second equation of said four equations defines a force equilibrium condition, a third equation of said four equations defines a moment equilibrium condition, and a fourth equation of said four equations defines a stress continuity condition.

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10. A method of analyzing structural stress as claimed in claim 8 wherein said force equilibrium condition is as follows:

$$\text{where } \sigma_m = \frac{1}{t_1} \int_0^{t_1} \sigma_x(y) dy + \frac{1}{t_1} \int_0^\delta \tau_{yx}(x) dx.$$

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$$(\sigma_M - \sigma_B)t_1\left(t - \frac{t_1}{2}\right) + \sigma_B t_1\left(t - \frac{t_1}{3}\right) + \sigma_m \frac{(t - t_1)^2}{2} + \sigma_b \frac{(t - t_1)^2}{6} = \sigma_m \left(\frac{t^2}{2}\right) + \sigma_b \left(\frac{t^2}{6}\right)$$

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$$\left(\frac{t_1^2}{2}\right)\sigma_m + \left(\frac{t_1^2}{6}\right)\sigma_b = \int_0^{t_1} \sigma_x(y)ydy + \delta \int_0^{t_1} \tau_{xy}(y)dy + \int_0^{\delta} \sigma_y(x)xdx$$

where δ represents a value derived from a representation of said structure and $\tau_{xy}(y)$ represents a through-thickness shear stress distribution of said structure, as determined from said representation of said structure.

12. A method of analyzing structural stress as claimed in claim 8 wherein σ_m' and σ_b' comprise respective sub-components of said structural stress σ_s and wherein said stress continuity condition at $y = t - t_1$ is as follows:

$$\sigma_M - \sigma_B = \sigma_m' + \sigma_b'.$$

13. A method of analyzing structural stress as claimed in claim 4 wherein said selected cross section of said structure is characterized by a thickness t and defines a non-monotonic through thickness stress distribution characterized by a minimum axial stress along a secondary axis lying in said localized region a distance t_2 below a surface of said structure, and wherein said method comprises:

determining said first and second components σ_M, σ_B of said structural stress σ_s in said localized region by performing an operations having results substantially equivalent to a result of solving simultaneously first and second equations with two unknowns, σ_M and σ_B .

14. A method of analyzing structural stress as claimed in claim 13 wherein:

said structural stress σ_s is calculated by combining said first component σ_M of said structural stress and said second component σ_B of said structural stress with two additional structural stress parameters σ_m' and σ_b' ; and

5 said additional structural stress parameters σ_m' and σ_b' are related to said first and second components σ_M , σ_B as follows:

$$\sigma_M - \sigma_B = \sigma_m' - \sigma_b'.$$

15. A method of analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said method comprising:

identifying local elements for structural stress extraction, wherein said local elements lie adjacent to said localized fatigue-prone region;

determining nodal force and moment vectors for said local elements;

15 converting selected ones of said nodal force and moment vectors to sectional force vectors n and moment vectors m with an appropriate mapping function, wherein said mapping function is selected such that said sectional force vector n has units of force per unit length and said sectional moment vector m has units of moment per unit length; and

calculating said structural stress utilizing the following equation

$$\sigma_s = \sigma_B + \sigma_M$$

where σ_B is proportional to said sectional moment vector m and σ_M is proportional to said sectional force vector n .

16. A method of analyzing structural stress as claimed in claim 15 wherein said structural stress is calculated utilizing the following equation

$$\sigma_s = \sigma_B + \sigma_M = \frac{12mz}{t^3} + \frac{n}{t}$$

where t corresponds to the thickness of said structure in said fatigue-prone region and

$$-(t/2) \leq z \leq +(t/2).$$

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17. A method of analyzing structural stress as claimed in claim 15 wherein said nodal force and moment vectors are retrieved directly from a finite element model of said structure.

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18. A method of analyzing structural stress as claimed in claim 15 wherein said nodal force and moment vectors are determined by generating a stiffness matrix for said local element from a finite element model of said structure and computing said nodal force and moment vectors from said stiffness matrix.

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19. A method of analyzing structural stress as claimed in claim 15 wherein said nodal force and moment vectors are determined by:

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generating from a finite element model of said structure stiffness matrices and nodal displacements for the local elements in said fatigue-prone region;

multiplying said stiffness matrices and said nodal displacements to obtain global nodal force and moment vectors at nodal points of said local elements; and

transforming global force and moment vectors from the global coordinate system to the local coordinate system of an element of interest.

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20. A method of analyzing structural stress as claimed in claim 15 wherein said nodal force and moment vectors are determined by transforming stiffness matrices and nodal displacements to global and local coordinates and solving for force and moment vectors of said local element.

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21. A method of analyzing structural stress as claimed in claim 20 wherein said global coordinates of said stiffness matrices and nodal displacements are used to determine nodal force and moment vectors of said local element of interest.

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22. A method of analyzing structural stress as claimed in claim 15 wherein said fatigue-prone region includes a weld and wherein said mapping function is selected according to characteristics of said weld.

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23. A method of analyzing structural stress as claimed in claim 15 wherein said mapping function is a linear function.

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24. A method of analyzing structural stress as claimed in claim 23 wherein said mapping function defines a magnitude that progresses linearly from a minimum value to a maximum value between adjacent nodes of said local element.

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25. A method of analyzing structural stress as claimed in claim 24 wherein said mapping function is a non-linear function.

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26. A method of analyzing structural stress as claimed in claim 24 wherein said mapping function is a bi-linear mapping function comprising a combination of a first

linear mapping function from a first node N_1 to a virtual node N_3 and a second linear function from said virtual node N_3 to a second node N_2 .

- 5 27. A method of analyzing structural stress σ_s in a localized fatigue-prone region of a structure from a three-dimensional finite element solid model of the structure, said method comprising:

identifying a group of elements for structural stress extraction, wherein said local elements lie adjacent to said localized fatigue-prone region;

10 determining nodal forces for said local elements from said finite element solid model of said structure;

converting selected ones of said nodal force vectors to equivalent sectional forces and moments along a selected cross section including said localized fatigue-prone region; and

15 calculating said structural stress utilizing the following equation

$$\sigma_s = \sigma_B + \sigma_M = \frac{12mz}{t^3} + \frac{n}{t}$$

where m comprises a sectional moment vector, n comprises a sectional force vector, t corresponds to the thickness of said structure in the fatigue-prone region, and z ranges from $+t/2$ at a top surface of said structure to $-t/2$ at a bottom surface of said structure.

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28. A method of analyzing structural stress σ_s in a localized fatigue-prone region of a structure from a three-dimensional finite element solid model of the structure, said method comprising:

- 25 identifying at least one local element for structural stress extraction, wherein said local element lies adjacent to said localized fatigue-prone region;

determining stress resultants $f_{x'}$, $f_{z'}$, and $m_{y'}$ for said local element from said finite element solid model of said structure, wherein said stress resultants represent the sectional forces and moments for said local element; and

calculating structural stress σ_s in said localized fatigue-prone region utilizing the following equation

$$\sigma_s = \sigma_M + \sigma_B = \frac{f_{x'}}{t} + \frac{6(m_{y'} + \delta f_{z'})}{t^2}$$

where δ and t represent dimensional values of said local element.

29. A technique for analyzing structural stress σ_s in a localized fatigue-prone region of a structure defining a substantially monotonic through thickness stress distribution, said technique comprising:

measuring displacement in the vicinity of said fatigue-prone region by configuring at least one measuring device to

measure displacement along a first cross section of said structure offset a distance L from said fatigue-prone region, and

measure displacement along a second cross section of said structure offset a distance $L-l$ from said fatigue prone region;

decomposing stress measurements at said strain gauges as follows

$$\sigma_b^B = \frac{1}{2}(\sigma_T^B - \sigma_B^B)$$
$$\sigma_b^C = \frac{1}{2}(\sigma_T^C - \sigma_B^C)$$

where said superscript B corresponds to a measurement at said first cross section, said superscript C corresponds to a measurement at said second cross section, said

subscript **T** corresponds to a measurement at a top surface of said structure, and said subscript **B** corresponds to a measurement at a bottom surface of said structure; and approximating structural stress σ_s in said fatigue-prone region as follows

$$\sigma_s = \sigma_T^C + \frac{L}{l}(\sigma_b^C - \sigma_b^B)$$

5 where the distances **L** and **l** are measured in terms of fractions of thickness **t**.

30. A technique for analyzing structural stress σ_s in a localized fatigue-prone region of a structure defining a non-monotonic through thickness stress distribution, said technique comprising:

measuring displacement in the vicinity of said fatigue-prone region by configuring at least one measuring device to

measure displacement along a first cross section of said structure offset a distance **L** from said fatigue-prone region,

measure displacement along a second cross section of said structure offset a distance **L-l** from said fatigue prone region, and

measure displacement along a third cross section of said structure, wherein said third cross section defines an approximately linear structural stress distribution;

20 decomposing stress measurements at said strain gauges as follows

$$\sigma_b^B = (\sigma_T^B - \sigma_m)$$

$$\sigma_b^C = (\sigma_T^C - \sigma_m)$$

where said superscript **B** corresponds to a measurement at said first cross section, said superscript **C** corresponds to a measurement at said second cross section, said

subscript T corresponds to a measurement at a top surface of said structure, and σ_m is a through thickness mean stress measurement taken at said third cross section; and approximating structural stress σ_s in said fatigue-prone region as follows

$$\sigma_s = \sigma_T^C + \frac{L}{l} (\sigma_b^C - \sigma_b^B)$$

5 where the distances L and l are measured in terms of fractions of thickness t .

31. A computer-readable medium encoded with a computer program for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said program being operative to:

determine a through-thickness stress distribution $\sigma_x(y)$ along a selected cross section of said structure, wherein said localized region lies adjacent to said selected cross section;

determine a membrane component σ_M and a bending component σ_B of said structural stress σ_s in said localized region from said through thickness stress distribution $\sigma_x(y)$, wherein said membrane and bending components comprise simple structural stress distributions that are equilibrium-equivalent to said through thickness stress distribution $\sigma_x(y)$; and

calculate said structural stress σ_s by combining said first component σ_M of said structural stress and said second component σ_B of said structural stress.

32. A computer-readable medium encoded with a computer program for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said program being operative to:

identify local elements for structural stress extraction, wherein said local elements lie adjacent to said localized fatigue-prone region;
determin nodal force and moment vectors for said local elements;
convert selected ones of said nodal force and moment vectors to sectional force
5 vectors \mathbf{n} and moment vectors \mathbf{m} with an appropriate mapping function, wherein said mapping function is selected such that said sectional force vector \mathbf{n} has units of force per unit length and said sectional moment vector \mathbf{m} has units of moment per unit length; and
calculate said structural stress utilizing the following equation

$$\sigma_s = \sigma_B + \sigma_M$$

where σ_B is proportional to said sectional moment vector \mathbf{m} and σ_M is proportional to said sectional force vector \mathbf{n} .

33. A computer-readable medium encoded with a computer program for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said program being operative to:

identify a group of elements for structural stress extraction, wherein said local elements lie adjacent to said localized fatigue-prone region;

determine nodal forces for said local elements from said finite element solid model of said structure;

convert selected ones of said nodal force vectors to equivalent sectional forces and moments along a selected cross section including said localized fatigue-prone region; and

calculate said structural stress utilizing the following equation

$$\sigma_s = \sigma_B + \sigma_M = \frac{12mz}{t^3} + \frac{n}{t}$$

where \mathbf{m} comprises a sectional moment vector, \mathbf{n} comprises a sectional force vector, t corresponds to the thickness of said structure in the fatigue-prone region, and z ranges from $+t/2$ at a top surface of said structure to $-t/2$ at a bottom surface of said structure.

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34. A computer-readable medium encoded with a computer program for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said program being operative to:

identify at least one local element for structural stress extraction, wherein said local element lies adjacent to said localized fatigue-prone region;

determine stress resultants $f_{x'}$, $f_{z'}$, and $m_{y'}$ for said local element from said finite element solid model of said structure, wherein said stress resultants represent the sectional forces and moments for said local element; and

calculate structural stress σ_s in said localized fatigue-prone region utilizing the following equation

$$\sigma_s = \sigma_M + \sigma_B = \frac{f_{x'}}{t} + \frac{6(m_{y'} + \delta f_{z'})}{t^2}$$

where δ and t represent dimensional values of said local element.

20 35. A system for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said system including a controller programmed to:

determine a through-thickness stress distribution $\sigma_x(y)$ along a selected cross section of said structure, wherein said localized region lies adjacent to said selected cross section;

25 determine a membrane component σ_M and a bending component σ_B of said structural stress σ_s in said localized region from said through thickness stress

distribution $\sigma_x(y)$, wherein said membrane and bending components comprise simple structural stress distributions that are equilibrium-equivalent to said through thickness stress distribution $\sigma_x(y)$; and

calculate said structural stress σ_s by combining said first component σ_M of said structural stress and said second component σ_B of said structural stress.

36. A system for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said system including a controller programmed to:

identify local elements for structural stress extraction, wherein said local elements lie adjacent to said localized fatigue-prone region;
determine nodal force and moment vectors for said local elements;
convert selected ones of said nodal force and moment vectors to sectional force vectors n and moment vectors m with an appropriate mapping function, wherein said mapping function is selected such that said sectional force vector n has units of force per unit length and said sectional moment vector m has units of moment per unit length; and

calculate said structural stress utilizing the following equation

$$\sigma_s = \sigma_B + \sigma_M$$

where σ_B is proportional to said sectional moment vector m and σ_M is proportional to said sectional force vector n .

37. A system for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said system including a controller programmed to:

identify a group of elements for structural stress extraction, wherein said local elements lie adjacent to said localized fatigue-prone region;

determine nodal forces for said local elements from said finite element solid model of said structure;

convert selected ones of said nodal force vectors to equivalent sectional forces and moments along a selected cross section including said localized fatigue-prone region; and

calculate said structural stress utilizing the following equation

$$\sigma_s = \sigma_B + \sigma_M = \frac{12mz}{t^3} + \frac{n}{t}$$

where m comprises a sectional moment vector, n comprises a sectional force vector, t corresponds to the thickness of said structure in the fatigue-prone region, and z ranges from $+t/2$ at a top surface of said structure to $-t/2$ at a bottom surface of said structure.

38. A system for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said system including a controller programmed to:

identify at least one local element for structural stress extraction, wherein said local element lies adjacent to said localized fatigue-prone region;

determine stress resultants $f_{x'}$, $f_{z'}$, and $m_{y'}$ for said local element from said finite element solid model of said structure, wherein said stress resultants represent the sectional forces and moments for said local element; and

calculate structural stress σ_s in said localized fatigue-prone region utilizing the following equation

$$\sigma_s = \sigma_M + \sigma_B = \frac{f_{x'}}{t} + \frac{6(m_{y'} + \delta f_{z'})}{t^2}$$

where δ and t represent dimensional values of said local element.

39. A system for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said system comprising:

at least one measuring device configured to measure displacement in the vicinity of said fatigue-prone region by

measuring displacement along a first cross section of said structure offset a distance L from said fatigue-prone region, and
measuring displacement along a second cross section of said structure offset a distance $L-l$ from said fatigue prone region; and

a controller programmed to

decompose stress measurements at said strain gauges as follows

$$\sigma_b^B = \frac{1}{2}(\sigma_T^B - \sigma_B^B)$$
$$\sigma_b^C = \frac{1}{2}(\sigma_T^C - \sigma_B^C)$$

where said superscript B corresponds to a measurement at said first cross section, said superscript C corresponds to a measurement at said second cross section, said subscript T corresponds to a measurement at a top surface of said structure, and said subscript B corresponds to a measurement at a bottom surface of said structure, and

approximate structural stress σ_s in said fatigue-prone region as follows

$$\sigma_s = \sigma_T^C + \frac{L}{l}(\sigma_b^C - \sigma_b^B)$$

where the distances L and l are measured in terms of fractions of thickness t .

- 5 40. A system for analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said system comprising:

at least one measuring device configured to measure displacement in the vicinity of said fatigue-prone region by

10 measuring displacement along a first cross section of said structure offset a distance L from said fatigue-prone region,

 measuring displacement along a second cross section of said structure offset a distance $L-l$ from said fatigue prone region, and

15 measuring displacement along a third cross section of said structure, wherein said third cross section defines an approximately linear structural stress distribution; and

a controller programmed to

decompose stress measurements at said strain gauges as follows

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$$\sigma_b^B = (\sigma_T^B - \sigma_m)$$

$$\sigma_b^C = (\sigma_T^C - \sigma_m)$$

where said superscript B corresponds to a measurement at said first cross section, said superscript C corresponds to a measurement at said second cross section, said subscript T corresponds to a measurement at a top surface of said structure, and σ_m is a through thickness mean stress measurement taken at said third cross section; and

approximate structural stress σ_s in said fatigue-prone region as follows

$$\sigma_s = \sigma_T^C + \frac{L}{l} (\sigma_b^C - \sigma_b^B)$$

where the distances L and l are measured in terms of fractions of thickness t .

41. A method of analyzing structural stress in a localized fatigue-prone region of a structure, said method comprising:

determining a through-thickness stress distribution along a selected cross section of said structure, wherein said localized region lies adjacent to said selected cross section and said through-thickness stress distribution is determined from a finite element model of said structure;

determining a membrane component and a bending component of said structural stress in said localized region from said through thickness stress distribution, wherein said membrane and bending components comprise simple structural stress distributions that are equilibrium-equivalent to said through thickness stress distribution; and

calculating said structural stress by combining said first component of said structural stress and said second component of said structural stress, wherein said membrane component and said bending component are determined such that said structural stress calculation is substantially independent of the size, shape and distribution of mesh elements defining said finite element model of said structure.

42. A method of analyzing structural stress as claimed in claim 41 wherein said membrane component and said bending component are determined such that said structural stress is substantially independent of mesh size over a mesh size range

extending from about $0.16t$ and $0.1t$, along respective x and y axes, to about $2t$ and t along respective x and y axes, where t represents the thickness of said structure.

5 43. A method of analyzing structural stress as claimed in claim 41 wherein said membrane component and said bending component are determined such that said structural stress is substantially independent of mesh size over a mesh size range extending from about $0.008t$ and $0.02t$, along respective x and y axes, to about $0.4t$ and $0.5t$ along respective x and y axes.

10 44. A method of analyzing structural stress as claimed in claim 41 wherein said membrane component and said bending component are determined such that said structural stress is substantially independent of mesh size over a mesh size range
15 extending up to about $2t$ and t along respective x and y axes.

20 45. A method of analyzing structural stress σ_s in a localized fatigue-prone region of a structure, said method comprising:

determining a through-thickness stress distribution $\sigma_x(y)$ along a selected cross section of said structure, wherein said localized region lies adjacent to said selected cross section;

determining a membrane component σ_M and a bending component σ_B of said structural stress σ_s in said localized region from said through thickness stress
25 distribution $\sigma_x(y)$, wherein said membrane and bending components comprise simple structural stress distributions that are equilibrium-equivalent to said through thickness stress distribution $\sigma_x(y)$; and

46. A method of analyzing structural stress as claimed in claim 45 wherein said membrane component σ_M and said bending component σ_B are determined as a function of the following variables:

a thickness variable t corresponding to a thickness of said structure in said selected cross section.

a distance variable y corresponding to a distance from $y=0$ to a material point of interest along said selected cross section;

a finite element value δ representing a value defined in a finite element representation of said structure; and

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48. A computer-readable medium encoded with a computer program for calculating structural stress σ_s in a localized fatigue-prone weld region of a structure from a finite element model of said structure, said program being operative to:

identify local elements for structural stress extraction, wherein said local elements lie adjacent to said weld region;

determine nodal displacements and nodal force and moment vectors for said local elements from said finite element model;

convert selected ones of said nodal force and moment vectors to sectional force vectors n and moment vectors m , wherein said conversion is performed in a work equivalent manner with respect to said nodal displacements determined for said nodal force and moment vectors; and

calculate said structural stress from said sectional force vectors n and moment vectors m .

49. A system for calculating structural stress σ_s in a localized fatigue-prone weld region of a structure from a finite element model of said structure, said system including a controller programmed to:

identify local elements for structural stress extraction, wherein said local elements lie adjacent to said weld region;

determine nodal displacements and nodal force and moment vectors for said local elements from said finite element model;

convert selected ones of said nodal force and moment vectors to sectional force vectors n and moment vectors m , wherein said conversion is performed in a work equivalent manner with respect to said nodal displacements determined for said nodal force and moment vectors; and

calculate said structural stress from said sectional force vectors n and moment vectors m .